Prospects of Numerical Full-Wave Techniques in Telecommunication Channel Modelling

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***Abstract*—In telecommunication channel modelling the wavelength is small compared to the physical features of interest, therefore deterministic ray tracing techniques provide solutions that are more efficient, faster and still within time constraints than current numerical full-wave techniques. Solving fundamental Maxwell's equations is at the core of computational electrodynamics and best suited for modelling electrical field interactions with physical objects where characteristic dimensions of a computing domain is on the order of a few wavelengths in size. However, extreme communication speeds, wireless access points closer to the user and smaller pico and femto cells will require increased accuracy in predicting and planning wireless signals, testing the accuracy limits of the ray tracing methods. The increased computing capabilities and the demand for better characterization of communication channels that span smaller geographical areas make numerical full-wave techniques attractive alternative even for larger problems. The paper surveys ways of overcoming excessive time requirements of numerical full-wave techniques while providing acceptable channel modelling accuracy for the smallest radio cells and possibly wider. We identify several research paths that could lead to improved channel modelling, including numerical algorithm adaptations for large-scale problems, alternative finite-difference approaches, such as meshless methods, and dedicated parallel hardware, possibly as a realization of a dataflow machine.**

***Index Terms*—Radio wave propagation, far-field computation, signal prediction, full wave methods, numerical methods.**

I. INTRODUCTION

COMPREHENSIVE understanding of radio wave propagation is essential to any further development of wireless networks. Ultra-dense ultra-reliable and low latency communications of the 6G vision, massive antenna systems, highly dynamic mobility, location aware communications, widespread use of artificial intelligence, shift toward higher frequency bands—all these aspects and concepts require better knowledge of radio wave propagation. The topic has already been a subject of long-term research since the appearance of wireless communications. The empirical models are still pervasive in wireless coverage planning; however, they do not allow modelling of channel spatial and temporal characteristics in the required levels of details (MIMO, UWB, DSSS, OFDM). More advanced deterministic models that include only a subset of the propagation environment elements are of limited use as well and do not guarantee the continued increase of wireless systems data rates, throughput and reliability.

Multipath propagation leads to signal delay spread. Direction of arrival is important for antenna systems. These and similar parameters are readily available in advanced deterministic models, which take into account detailed knowledge of the environment geometry. Over the past decade, models based on Geometric Optic (GO) have become popular for electromagnetic wave propagation prediction. Deterministic in nature, ray tracing algorithms can be only conditionally characterized as full-wave techniques if they do not include some sort of calibration to measurements. However, high-frequency approximation, where rays mimic narrow beams of light, has significant deviations from radio frequency diffraction and scattering behavior [1], [2]. Geometric theory of diffraction improves the accuracy to a certain degree. On the other hand, scattering has significant impact on indoor propagation but no appropriate ray tracing approximation. In general, any ray tracing extension either brings significant processing burden, limiting its use to small geometries or requires two-dimensional simplification.

On the other hand, when the problem is in the order of tens of wavelengths, modelling based on numerical full-wave techniques dominates. The full-wave techniques are centered on Maxwell's partial differential equations of electrodynamics, which represent one of the most outstanding achievements of the 19th century physics. The unification of electric and magnetic field by four equations and the prediction of electromagnetic waves were undoubtedly the breakthrough points in science. Numerical solutions to these equations are fundamental tool in the development of electronics, communication devices, computers, lasers, antenna systems and many other fields with problems of similar scale. Further, the last decade has seen introduction of full-wave techniques in the classical telecommunication modelling, which goes beyond several hundred wavelengths limit. Most of the proposals are constrained to two-dimensional geometries with many simplifications due to the extraordinary computing demands.

Finite-difference time-domain (FDTD) methods and many other numerical full-wave techniques encapsulate all interactions of electromagnetic waves with matter, such as refraction, reflection, diffraction or scattering, with computational complexity being independent of the effect, its direction or the number of repetitions. This cannot be said for all analytical approaches, including many full-wave techniques. For example, integral equations cannot handle atmospheric conditions or urban architecture [3]–[5]. The use of parabolic equations is limited by the propagation angle from the paraxial direction [6]. Atmospheric conditions also cannot be accounted for

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in already mentioned ray tracing techniques, which have in general difficulties modelling curved surfaces, terrain profile or consequent diffraction phenomena [7].

Acceleration of the full-wave techniques for their greater acceptability in electrically large problems is an alternative to accuracy improvements of existing empirical or deterministic models. Such an approach would require further adaptations of numerical methods that fully account for the propagation effects and rigorously capture the physics of wireless links. The paper surveys ways of overcoming excessive time requirements of numerical full-wave techniques while providing acceptable channel modelling accuracy for the smallest radio cells and possibly wider. Our attention is on complex geometries with many materials. Three different research paths are identified: numerical algorithm adaptations for large-scale problems, alternative finite-difference approaches, such as meshless methods, and dedicated parallel hardware, including dataflow machines.

In the following, Section II gives a short overview of traditional channel modelling, including stochastic, empirical and ray-tracing models. Finite-difference time-domain methods are studied in Section III. Non-FDTD techniques are discussed in Section IV. Hardware acceleration of numerical procedures can, at least to some extent, scale the size of viable problems. The review of approaches and related problems is given in Section V. We briefly discuss the subject of accurate environment modelling in Section VI. The prospective concepts for wider use of numerical full-wave techniques in telecommunication channel modeling are summarized in Section VII, followed by the conclusion in Section VIII.

II. TRADITIONAL CHANNEL MODELLING

Telecommunication channel modelling has a rich history with numerous literature references. Chronologically, the modelling techniques progressed from stochastic and empirical models towards highly deterministic ray tracing approaches.

A. *Stochastic and Empirical Models*

General overview of the radio propagation models can be found in COST 273 report [8]. Stochastic models are based on the expected signal averages in distinct classes of environments. On the other hand, empirical models aim at more diverse propagation environments [9], [10]. They approximate radio channels by parametric functions based on extensive measurements and generally include path loss expression with an environment-specific path loss exponent [11]. Well known in this category are Ikegami, Wallfisch and Hata models [12]–[14]. Time domain characteristics, the RMS delay spreads and the angle of arrival have been integrated in the models by Saleh and Valenzuela [15]. Short computation time of the empirical models is offset by large prediction errors, especially in the heterogeneous environments.

B. *Geometrical Optics*

Ray tracing is highly deterministic channel modelling, as opposed to the empirical approach. It allows advanced channel characteristics evaluation, such as delay spread or direction of arrival, at the cost of higher processing efforts. Based on the principles of geometrical optics, the method effectively traces a large number of rays from the transmitting source in all directions into the scene. The concept of a reception sphere is usually needed to detect rays passing by the receivers [16], [17]. The algorithms from this group refer to the principle as ray launching [18], ray shooting and bouncing (SBR) [19], pincushion method [20] or more elaborated ray-tube [21] and beam tracing [22], the latter aggregating rays to reduce computational complexity and effectively converging to the second approach, known as the method of images [23]. Hybrid methods [24] and Gaussian beam tracking [25] are building on the further improvements of image theory.

The common denominator of all ray tracing algorithms is high frequency approximation of propagating waves, where a single ray mimics behavior of a thin beam of light. The simplification most notably affects the accuracy of diffraction modelling, which is negligible for many practical purposes at optical frequencies [26]. Geometrical Theory of Diffraction (GTD) introduces diffracted rays in order to approximate Maxwell's equations at the edge of two conducting half-planes [1] with obvious discontinuity between the incident and reflected shadow areas. The Uniform Theory of Diffraction (UTD), proposed by Kouyoumjian and Pathak [2], makes the transition smoother. The UTD has been afterwards extended to handle diffraction edges with finite conductivity [27], [28].

The discrete nature of rays with no thickness shows a weakness in the aggregation step of general ray tracers where nearby rays to the observation point need to be differentiated based on the sequences of previously encountered interactions. The space- and time-consuming task constrains the algorithm either to smaller geometries or to channel modelling with a significant number of double-counting errors. In previous work, the author proposed Bloom filters configured with marginal false-positive rate as a replacement of the exact wavefront differentiation with substantial computation gains [7].

Accurate ray tracing in indoor environments is predicated on the appropriate handling of diffuse scattering [17], [29]–[31]. Geometrical optics has no satisfying solution to the problem. Several new approaches [29], [31] try to alleviate the shortcoming, all by significantly increasing processing load and running time [17].

Ray tracing is considered frequency domain approach. On the other hand, Time Domain Geometrical Optics (TDGO) is promising alternative for wideband simulations [32]; however the method has not achieved as much attention as its frequency counterpart.

III. FINITE-DIFFERENCE TIME-DOMAIN METHODS

The Finite-Difference Time-Domain (FDTD) method is arguably the simplest full-wave technique. It was proposed by Kane Yee, who discretized Ampere's and Faraday's laws by the second-order central differences in 1966 [33]. The method is an explicit finite difference method. For example, the Faraday's law for the propagation in one-dimensional space along x-direction simplifies to

$$-\mu\frac{\partial H_y}{\partial t} = -\hat{a}_y\frac{\partial E_z}{\partial x}, \tag{1}$$

where μ is permeability and $\hat{a}_y$ denotes a unit vector pointing in y-direction. On the other hand, Ampere's law can be written as

$$\varepsilon\frac{\partial E_z}{\partial t} = \hat{a}_z\frac{\partial H_y}{\partial x}, \tag{2}$$

with ε being permittivity of propagation space and $\hat{a}_z$ a unit vector pointing in the direction of a magnetic field. Central-difference approximation gives rise to discrete equations

$$\begin{aligned} H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right] =& H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2}\right] \\ &+ \frac{\Delta_t}{\mu\Delta_x}\left(E_z^q\left[m+1\right] - E_z^q\left[m\right]\right) \end{aligned} \tag{3}$$

and

$$\begin{aligned} E_z^{q+1}\left[m\right] =& E_z^q\left[m\right] \\ &+ \frac{\Delta_t}{\varepsilon\Delta_x}\left(H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right] - H_y^{q+\frac{1}{2}}\left[m-\frac{1}{2}\right]\right), \end{aligned} \tag{4}$$

where Δ_x is the spatial grid distance and Δ_t is the time interval between computations at the same spatial location. The field values are indexed in space by m and in time by q.

The Courant-Friedrich-Levy (CFL) restriction of space to time ratio is mainly to blame for the high computational demands. Hence, the first applications were bounded to electrically small problems. The simplicity of a single computation step, which depends only on the values of immediate neighbors, is affected by complex boundary conditions. The simplest Absorbing Boundary Condition (ABC) is formulated on the notion of perfect impedance matching. The ABCs typically require several layers of specialized nodes in space [34]–[39]. Bounding conditions are active field of research with numerous proposals in the last several decades.

Error sources of the finite-difference full-wave techniques are well understood and mathematically explained. High computational complexity and progressive accumulation of errors with increasing propagation distance are the fundamental limitations. That is, numerical solutions either are inherently approximate due to the computer finite precision or because of approximations needed to derive numerical algorithm. Iterations lead to accumulation of delay or phase errors, which show as nonphysical phenomena, such as anisotropy, broadening and ringing of pulses, imprecise wave cancellations and virtual refractions. The exception is one-dimensional variant of the problem, where under proper conditions exact computation is possible. Numerical dispersion can be reduced to any degree by finer computational grid; however, this reduction has limited practical value. Discretization artefacts disappear only in the limit with finer meshes providing mostly theoretical way to control the error in large-scale simulations. Time limitations quickly prevent computation in any reasonable time frame. Therefore, numerical acceleration and balanced error handling are two key tasks that need to be faced while addressing problem sizes found in wireless communications. Numerical dispersion and instability are not the consequence of calculations in finite precision, but rather property of the finite difference approximation.

A. Handling Large-Scale Numerical Dispersion

Numerous proposals exist to alleviate dispersion problem. For example, the increase in permittivity and permeability helps to shift phase-velocity curve in a way to fit better average signal propagation, thus reducing dispersion problem for narrowband simulations. Dispersion depends on frequency as well as on propagation direction, i.e., anisotropy, and basically lowers phase velocity. Using fourth-order differences for the spatial first-derivatives to implement the curl operator significantly reduces dispersion at the cost of increased processing time [40]. However, in this case, material discontinuities require special boundary conditions. In [41] a smooth continuous function is used to reduce discontinuity problem. Alternative to the above techniques is to modify lattice geometry. Anisotropy significantly reduces if regular hexagonal grid is used in two dimensions [42] or, when extended to three dimensions, more complex tetra decahedron/dual-tetrahedron mesh. Leading second-order error term compensates across the grid, thus placing hexagonal grid on par with the fourth-order spatial algorithms, while keeping dependence exclusively on neighboring nodes and avoiding already mentioned problem of material discontinuities. Use of hexagonal grids in telecommunication geometries is still largely work-in-progress.

On the other hand, meshless alternatives with arbitrarily distributed computation nodes [43]–[45] are largely at the proof-of-concept stage. In principle, numerical methods based on regular grids have meshless alternative with better modelling of irregular geometries and arbitrarily distribution of computation nodes. The topic of meshless differential solvers is highly developed for a number of fields in physics, such as heat convection and hydrodynamic. The mesh-free smoothed particle hydrodynamics (SPH) method has already been applied to the electromagnetic modelling as well. The baseline concept relies on the representation of a function as a volume integral of that function multiplied by the Dirac delta pulses. In approximation, Dirac delta function is replaced by the so-called smoothing kernel function, i.e., kernel approximation. If the kernel is an even function, its volume integral is unity, in the limit converges toward Dirac delta and is compact, then the approximation is of second order accuracy in terms of the smoothing length. Kernel approximation simplifies calculation of spatial derivatives, which translates to kernel function differentiation. By replacing the integral with a sum, infinitesimal volumes are changed to discrete volumes, i.e., particle approximation. The above principles were already used to evaluate spatial steps, while time stepping remains as in Yee algorithm. High computing complexity of kernel approximation and the need to access more than just immediate neighbors raise some concerns about the method viability for large geometries, which should be properly addressed in the future.

B. Coping with Instability

An attempt to accelerate simulations by using larger time steps in large geometries is thwarted by numerical instability of the baseline algorithm. In practice, instability shows as exponentially growing fast oscillations. Classical Yee algorithm is constrained by the temporal to spatial step ratio, which is commonly known as the CFL limit. The problem geometry confines the spatial step, which through CFL ratio fixes the time step. The total number of time steps follows from the required observation time and has tendency to expand quickly out of acceptable boundaries. Alternating-Direction-Implicit FDTD (ADI FDTD) has offered a way to overcome the Courant limit already in 1980's, but only for problems having a cell size much smaller than the observed wavelength [46]–[49]. The authors proposed collocated electrical and magnetic fields in time, as opposed to being staggered, and provided explicit tridiagonal matrix systems that are not constrained by the CFL limit. However, numerical dispersion is still present and keeps growing even above the CFL limit. The time step is independent from the spatial step, but should be small enough to resolve the largest spectral components. The ADI FDTD is opening a new way of solving larger problems. Nevertheless, it discards fine-grained parallelism of FDTD. Similar approach is LOD-FDTD (Locally One-Direction FDTD) [50], [51]. R-FDTD (Reduced FDTD) addresses the problem of excessive memory requirements [52], but further increases processing time.

C. Large-Scale Applications

The use of FDTD in telecommunication problems is today generally limited to two-dimensional modelling with a few three-dimensional attempts [49], [53], [54]. A combination of the unconditionally stable algorithm, the fourth-order space differences and the moving window has already been evaluated in the context of two-dimensional urban scenario with a 550 m cell radius [41]. The use of moving window with a pulse excitation source limits processing to the sliding area with the most energy. It is a promising way to accelerate simulations over electrically large domains, with examples including tunnels [55], modelling wave propagation over the ocean [56] (40 m in three dimensions), and the prediction of Loran-C 100 kHz Gaussian modulated pulse ground wave along 400 km long path [57].

IV. OTHER FULL-WAVE TECHNIQUES AND HYBRIDS

The integral form of Maxwell's equations serves as a foundation of a numerical solver with very limited applications to small indoor environments [58], [59]. The approach does not require explicit boundary conditions. Homogeneous dielectric or conductive geometries are solved by the surface integrals [60], whereas inhomogeneous materials require the use of volume integration [61]. The equations discretization is based either on Method of Moments (MoM) [60] or on hybrid Finite-Element Boundary-Integral (FE-BI) [61]. The computation can be accelerated for larger repetitive geometries by Array Decomposition-Fast Multipole Method (AD-FMM) [62]. Volume Electric Field Integral Equation (VEFIE) has been proposed for three-dimensional geometries, which can be accelerated either by approximate MultiLevel Fast Multipole Algorithm (MLFMA) or by the exact Conjugate Gradient-Fast Fourier Transform (CG-FFT) [58].

Expansion of the FDTD differential formulation in space with the Deslauriers-Dubuc biorthogonal interpolating functions leads to the Scaling Multi-Resolution Time Domain (S-MRTD) method [63]–[65]. Accuracy remains comparable to the FDTD but at lower spatial sampling, which results in up to 8 times acceleration [64]. However, only two-dimensional variant has been studied in indoor geometries.

Noteworthy is the generalization of the Transmission Line Matrix (TLM) method from the electrical circuit design. Its adaptation to modelling propagation in urban environments can be considered as a time-domain full-wave method [66]. The principle of flows implicitly models wave reflections and diffractions. High computational cost is slightly reduced in its multi-resolution frequency-domain variant MR-FDPF (Multi-Resolution Frequency Domain Parallel Flow) [67]. Simplifications in three-dimensional space by avoiding low-impact propagation modes or a combination of multiple two-dimensional sub problems [68] give approximate solutions that require calibration. Reducing computation burden by running simulations at lower frequencies also leads to approximations.

The base premise of the parabolic equation method is paraxial approximation. Modelled geometry should have a preferential propagation direction, with important physical phenomena not occurring at angles greater than 15 degrees from this direction. Tunnels and other special geometries are best suited for the parabolic equation [69]. Wider use is enabled by the relaxation of 15-degree constraint [70], [71]. Numerical solution is, as in FDTD, based on finite differences and time stepping. Narrow geometries are particularly appropriate, with results comparable to the ray tracing approaches. On the other hand, results more closely resemble those of empirical models when applied to general geometries [72].

The PSTD (PseudoSpectral Time-Domain) method embeds frequency analysis to evaluate spatial derivatives, but it remains time-domain method [73]. Spatial derivatives are evaluated exactly for at least two points per wavelength. Time derivatives still include the second-order error term. Perfectly matched layer ABC is required to prevent the Fast Fourier Transform (FFT) periodic behavior to generate additional errors. However, only geometries made entirely of dielectrics can be modelled by PSTD, because of the requirement of continuity of tangential fields, which is clearly violated on metal surfaces.

Hybrid methods are common compromise between speedier but less accurate ray tracing and computationally demanding full-wave techniques. The numerical treatment is performed in the areas with complex discontinuities or areas with the size of details close to the simulated wavelength, whereas ray tracing is used in the rest of geometry. The transition between areas poses a major challenge. Further, user interaction is needed to define area partitioning [58], [74], [75]. Due to complexities involved, hybrid approaches in three dimensions are even rarer [76], [77].

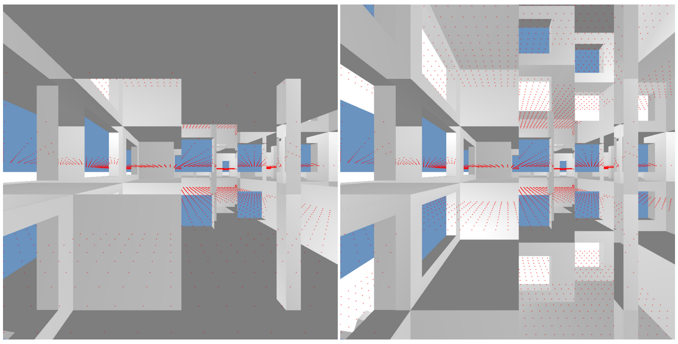

Fig. 1. Radio signal reflections and refractions are recursively traversed using graphic primitives. The illustration shows two snapshots of a framebuffer object at increasing reflection depth with black-and-white rendering of scene objects and visible reception points superimposed as red dots.

V. USE OF HARDWARE ACCELERATORS

Wider applicability of full-wave techniques and particularly the differential ones can be delivered, at least to some extent, by hardware acceleration. Gate level implementations, graphical accelerators and dataflow computing architectures have already been proposed in the context of large-scale propagation modelling; though they are more frequently applied to the smaller numerical full-wave problems and based on algorithms that depend only on a small number of neighboring cells. Solving Maxwell's equations numerically is generally well suited to extensive hardware parallelism. Similar trend is present in the more traditional channel modelling techniques. For instance, radio signal ray tracing is not much different from the established methods in computer graphics, which heavily rely on the hardware acceleration. In previous work, the author showed that even standard rendering can be used to accelerate radiofrequency channel modeling [78]. For example, Fig. 1 shows modelling of a reception point's visibility by rasterization after two consequent reflections.

A. GPU Acceleration

First papers on the dedicated computing architectures for the acceleration of numerical finite difference methods appeared shortly after the release of Compute Unified Device Architecture (CUDA) [79]. In 2009, FDTD implementations spanning entire clusters of CUDA supporting cards emerged [80]. GPU cluster managed to simulate 3 G cells with the throughput of up to 13 G cells/s. According to the report, 16-node Acceleware's G80 configurations achieved 29 times CPU cluster speed of comparable size. Optimal time step selection for improved precision on GPUs near stability limit is examined in [81]. The multi-resolution S-MRTD fits particularly well to GPU architecture [65] with speed up factor of 30, while the reference FDTD peaked at 10. The lack of further research for this method can be attributed to the implementation complexity of interpolating and pulse basis functions.

Numerous non-telecommunication problems that are being solved by the FDTD method have been successfully ported to GPUs, such as TEz-FDTD problem inside piecewise-linear recursive-convolution dispersive media [82], 3D FDTD human model for biomedical engineering [83], or the hybrid implicit-explicit FDTD for the analysis of printed circuit board shielding [84]. The almost unconditionally stable ADI FDTD algorithm, which is of particular importance for large-scale problems, has also been implemented on GPUs [85]. The efficiencies of fundamental ADI FDTD and of locally one-dimensional LOD FDTD on GPUs are discussed in [86]. Meshless time domain modelling of bended waveguide was numerically investigated on GPUs [87] as well.

The actual acceleration depends on the algorithm and the hardware. Still, not many proposals are targeting large-scale geometries while being capable of running in multiple dimensions. The reasons can be attributed to the high memory requirements and to the ever-present communication bottleneck between the processing cores and the memory. Furthermore, GPU parameters need to be carefully tuned for best performance.

B. Gate-Level Acceleration

High regularity and local dependencies make numerical full-wave techniques attractive for implementations on the gate level, e.g., in Field-Programmable Gate Arrays (FPGAs) or even in Application-Specific Integrated Circuits (ASICs). The finite resources of integrated circuits currently limit existing proposals to conceptual implementations, smaller or simplified variants of FDTD. Typical use of fixed-point arithmetic brings some reduction in accuracy in addition to the already mentioned memory bottleneck. Chip-level interconnection architectures should be investigated further in order to map larger problems on the available resources and to make better use of distributed memory schemes. Some attempts have been made to ease low-level coding by using Open Computing Language (OpenCL) or to increase the algorithms' abstraction level by assuming existence of embedded processor arrays. In the latter case Single Instruction, Multiple Data (SIMD) principles are the most promising.

Custom gate-level implementation of one-dimensional FDTD was already proposed in 2002 [88]. On the other hand, three-dimensional implementation of Yee algorithm appeared in 2003 [89]. The conceptual solver for resonant cavity problem with a small number of cells did not outperform a personal computer, but revealed limited accuracy due to fixed-point arithmetic and slow memory access. Detailed investigation of the fixed-point arithmetic establishes that acceptable error requires at least 28-bit numbers [90]. Single precision floating-point implementation is studied in [91]. The conclusion of the above initial works was that the accurate gate-level implementation is highly resource demanding.

Increasing memory bandwidth is also a priority of the FPGA architecture for the 3D FDTD based on Open CL in [92], [93], where FPGA design at 114 GFLOPS is reported. According to the authors, 4 times speed-up over GPU was achieved. Multi-core processor embedded in FPGA is studied with respect to power efficiency while solving FDTD in [94]. Finally, synchronous data transfers of FPGA SIMD array processor also minimize data transfer overhead in comparison to the asynchronous GPU architecture with additional speed-up benefits [95].

Memory bottleneck can be avoided to some extent by switching to dataflow algorithms. The research of the dataflow full-wave techniques can still be classified as being in the initial phase. For example, dataflow computer was suggested in [96]–[99]. Further, Maxeler dataflow extension boards were exploited in [100] with special attention paid to Dirichlet, periodic and absorbing boundary conditions.

C. *Prospects of Quantum Computing*

Substituting classical physics with quantum mechanics is believed by many to be the next revolution in computing, although the arguments of skeptics should not be overlooked [101]. The superposition property of quantum states, where the system with n qubits can be described by 2^n quantum amplitudes, is predicted to enable massive parallelism of a scale unseen in classical computing. The entanglement of quantum states will enable new forms of interactions and operations. The initial theoretical success in applying quantum computing to cryptanalysis was quickly replicated in other fields, such as computational chemistry and optimization research. With respect to the electromagnetic field modeling, a quantum algorithm was already developed that simulates the wave equation [102]. That led to the proposal of a quantum based TLM algorithm [103] for the simulations of electromagnetic structures. However, a practical quantum acceleration of full-wave techniques still requires a significant advance in the field, including overcoming barriers to physical integration. In long term, we believe that quantum technology has potential to cope with the enormous computational demands of a larger scale telecommunication channel modeling.

VI. ENVIRONMENT MODELLING

Increased accuracy of numerical methods is predicated on having indoor and outdoor geometries that capture significant level of details, going beyond simple wall and building models [104]. Additional data should be collected about the shape and the composition of the objects in the actual scenario, including indoor objects as small as furniture. Accurate 3D modelling of interior and exterior is becoming an important topic also in many other research fields, e.g., robot navigation, interactive visualization, indoor localization, etc. Digital modelling of entire cities has been underway for some time to support urban planning. Manual construction of simulation geometries based on floorplans and using the hands-on knowledge about the interior design is tedious and time-consuming task. Therefore, automatic environment reconstruction methods are gaining in popularity [105].

The technologies involved include Mobile Light Detection and Ranging (LiDAR), inertial navigation techniques, various indoor localization approaches and Global Navigation Satellite System (GNSS). Some of them have already been tested in automatic mapping of transportation infrastructure [106] or in autonomous vehicle driving. Remote sensing technologies that collect information without making a physical contact with the objects have several shortcomings which need to be addressed in the future. Mobile data capture is further hindered by missing regions, variable sampling density and noise. Robust shape detection from raw data in combination with filtering and substructure clustering is one of the approaches to recover missing data [107].

Indoor modelling requires even more accurate feature extraction including the identification of the object composition in order to choose appropriate electrical properties. Structural elements, such as doors, windows, walls, floors and ceilings, can be detected by their shape [105]. Detecting missing information due to occlusion can be partially achieved by the identification of structural elements [108]. Indoor spaces are filled with complex details. Furthermore, they are subject of frequent changes. The geometry extraction process from the 3D point cloud images, efficient data fusion from multiple sensors, accurate feature extraction and other topics are, due to complexities involved, major research challenges.

VII. SUMMARY OF PROSPECTIVE CONCEPTS

In order to make numerical full-wave techniques viable and more attractive to telecommunication channel modelling we need to overcome a number of obstacles (Table I). Among the major hurdles is extremely high computational complexity that requires some drastic simplifications and the use of dedicated computer architectures. Next, numerical dispersion and instability are well-known phenomena in smaller computation domains which get even more pronounced in larger geometries. In addition to known solutions, like permittivity and permeability corrections, high-order differences, or alternate lattice geometries, some new approaches need to be developed that would also be applicable in wideband scenarios. Meshless solvers are established numerical technique for many problems in physics which appears promising for telecommunication channel modelling.

Three-dimensional numerical problem solving is attractive but usually resource-prohibitive even in smaller scenarios. A combination of multiple two-dimensional computations seems a viable alternative in larger scenarios. Open spaces make the use of moving window where only areas with the most energy are resolved also an attractive approach. Note that rich multipath in indoor scenarios prohibits similar tactic.

Less accurate scenarios could benefit from running simulations at lower frequencies, by avoiding low-impact propagation modes or from taking advantage of particular geometries, e.g., using parabolic equation in narrow spaces. Possible approaches include new hybrid methods that resolve larger homogenous spaces by alternative algorithms.

From the algorithmic perspective, synchronous-data transfers of SIMD architectures and dataflow principles should be used as much as possible to alleviate memory bottleneck problems. Optimal chip-level interconnections, which were neglected in conceptual dedicated hardware due to highly simplistic test domains, should also get proper attention. On the other hand, quantum computing may significantly accelerate the full-wave modeling; however, there is no consensus among the researchers on the time frame.

It has to be emphasized that the discussed concepts are not necessarily mutually compatible and a subset should be chosen for particular telecommunication problem. Tests so

TABLE I
PROSPECTIVE CONCEPTS FOR LARGE-SCALE NUMERICAL COMPUTATIONS

Obstacle	Possible ways to proceed	
Computational complexity	Reducing computation load	Use of moving window (outdoor) Simulations at lower frequencies 2.5 dimensional simulations Use of meshless solvers Avoiding low-impact modes Taking advantage of geometry Hybrid trade-offs
	Use of dedicated hardware	GPUs, FPGAs, ASICs SIMD architectures Dataflow architectures Chip-level interconnect optimizations
	Quantum computing	Overcoming barriers to physical integration
Dispersion	Permittivity and permeability corrections High-order differences and smooth cont. functions Alternate lattice geometries including hexagonal grids Meshless solvers with systematically distributed nodes	
Instability	ADI-like approaches	
Accurate geometries	Use of LiDAR, inertial sensors, GNSS, localization techniques, image processing, machine learning	

far strongly supports numerical acceleration in hardware with prospects of expanding applicable geometry sizes at least by order of magnitude. Further, capturing accurate environment geometries is another aspect that should not be overlooked as well.

VIII. CONCLUSION

Telecommunication channel modelling is founded on the laws of electromagnetic interactions with matter and it should meet the requirements of today and future communication technologies. Acceleration of the numerical full-wave techniques for their greater acceptability in electrically large problems is an alternative to accuracy improvements of existing deterministic models, where ray tracing shows accelerating trend to replace empirical models. We propose such a different approach by adapting the methods that fully account for the propagation effects and rigorously capture the physics of wireless links. With that respect, ways of overcoming excessive time requirements of numerical full-wave methods while providing acceptable channel modelling accuracy for the smallest radio cells and possibly wider are becoming increasingly important. Researchers can follow several paths: numerical algorithm adaptations for large-scale problems, alternative finite difference approaches, such as meshless methods, and dedicated parallel hardware, including dataflow machines. Further, quantum computing shows some prospects in the field. Trade-off between speed and accuracy from the viewpoint of full-wave techniques as well as from the viewpoint of today's deterministic methods should be systematically assessed. The basic knowledge about the large-scale numerical solutions is still limited and further research is crucial. The ever-increasing number of access points and closely related reduction of the wireless cell's effective geographical area suggest viability of the numerical full-wave techniques for the telecommunication channel modelling.

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